Additional Appendix (not intended for publication)

A.1 Supplements for Welfare Analysis

In this appendix, we describe some details about the welfare analysis in Section 7.2, where the relation between IPR protection and social welfare is investigated. Focusing our analysis on the balanced growth path (hereafter, BGP), we assume that the degree of IPR protection is kept constant at δ and the economy is on the BGP at time t. In order to clarify the dependence of steady state variables on δ, we indicate their values by "$*$" and "(δ)"; e.g., $w^*(\delta)$, $\mu^*(\delta)$, ... etc.

Since the economy is on the BGP, the log of output at time $\tau \geq t$ is given by $\ln Y^*_\tau(\delta) = \ln Y^*_t(\delta) + \gamma^*_Y(\delta)(\tau - t)$, where $Y^*_t(\delta)$ is the present level of output and $\gamma^*_Y(\delta)$ is the growth rate of output on the BGP, defined by (24). Substituting this expression and $c_t = Y_t/L$ into (1) yields the utility of household as of time t:

$$U^*_t(\delta) = \sum_{\tau=t}^{\infty} \beta^\tau \ln\left(\frac{Y^*_\tau(\delta)}{L}\right) = \frac{1}{1-\beta}\left(\ln Y^*_t(\delta) + \frac{\beta}{1-\beta}\gamma^*_Y(\delta) - \ln L\right). \tag{37}$$

From the discussions in Section 3.1, the output of a monopolistic sector and a competitive sector are given by $1/\lambda w^*(\delta)$ and $1/w^*(\delta)$, respectively. Then we obtain the present level of output $Y^*_t(\delta)$ as follows:

$$\begin{aligned}
\ln Y^*_t &= (\ln\lambda)\int_0^1 q_{it}\, di + \int_0^1 \ln x_{it}\, di \\
&= (\ln\lambda)\int_0^1 q_{it}\, di + \mu^*(\delta)\ln(\lambda w^*(\delta)) + (1-\mu^*(\delta))\ln(1/w^*(\delta)) \\
&= (\ln\lambda)\int_0^1 q_{it}\, di - \mu^*(\delta)(\ln\lambda) - \ln w^*(\delta).
\end{aligned}\tag{38}$$

In the remainder of the appendix, we assume that b is sufficiently large that the rate of economic growth is maximized at $\delta^{\max}$. We focus on the welfare effect of IPR protection when the level of IPR protection is changed within the neighborhood of $\delta = \delta^{\max}$. Note that, since there is no leapfrogging in this case (see Panel 1 of Figure 5), the equilibrium number of researchers and the wage level is determined by (15). In the neighborhood of $\delta = \delta^{\max}$, the number of researchers is positive, and therefore condition (15) holds with equality. The condition can be written in terms of steady-state variables:

$$\frac{1-\pi\mu^*(\delta)}{L-(1-\mu^*(\delta))n^*(\delta)} = \beta\overline{V}(\delta)g(n^*(\delta)) = w^*(\delta). \tag{39}$$

From the first equality, we obtain:

$$\begin{aligned} 1-\pi\mu^*(\delta) =& \beta\overline{V}(\delta)\left[Lg(n^*(\delta)) - (1-\mu^*(\delta))G(n^*(\delta))\right] \\ =& \beta\overline{V}(\delta)\left[Lg(n^*(\delta)) - \delta\mu^*(\delta)\right], \end{aligned} \tag{40}$$

where identity $G(n^*) = g(n^*)n^*(\delta)$ and stationarity condition $(1-\mu^*(\delta))G(n^*(\delta)) = \delta\mu^*(\delta)$ are used. Solving (40) for $n^*(\delta)$ and substituting the result into (39) gives:

$$w^*(\delta) = L^{-1}\left(1 - \frac{1-\beta}{1-\beta+\beta\delta}\pi\mu^*(\delta)\right). \tag{41}$$

By substituting (41) into (38), the present level of output can be written in terms of $\mu^*(\delta)$ as follows:

$$\begin{aligned} \ln Y_t^*(\delta) =& -(\ln\lambda)\mu^*(\delta) - \ln(1-\beta+\beta\delta-(1-\beta)\pi\mu^*(\delta)) \\ & + \ln(1-\beta+\beta\delta) + \ln L + (\ln\lambda)\int_0^1 q_{it}\, di. \end{aligned} \tag{42}$$

The following examines whether relaxing IPR protection marginally from the growth-maximizing level, $\delta^{\max}$, improves or deteriorates the social welfare. Differentiating (37) with respect to δ gives:

$$\frac{dU_t^*(\delta)}{d\delta} = \frac{1}{1-\beta}\frac{d\ln Y_t^*(\delta)}{d\delta} + \frac{\beta}{(1-\beta)^2}\frac{d\gamma_Y^*(\delta)}{d\delta}. \tag{43}$$

Note that, since $\gamma_Y^*(\delta)$ is maximized at $\delta = \delta^{\max}$, its derivative at $\delta^{\max}$ is zero. Therefore, at $\delta = \delta^{\max}$, (43) can be written as:

$$\left.\frac{dU_t^*(\delta)}{d\delta}\right|_{\delta=\delta^{\max}} = \frac{1}{1-\beta}\left.\frac{d\ln Y_t^*(\delta)}{d\delta}\right|_{\delta=\delta^{\max}}, \tag{44}$$

where, from (42) and $(1-\beta)/\beta \equiv r$:

$$\frac{d\ln Y_t^*(\delta)}{d\delta} = -(\ln\lambda)\frac{d\mu^*(\delta)}{d\delta} + \frac{1}{r+\delta-r\pi\mu^*(\delta)}\left(r\pi\frac{d\mu^*(\delta)}{d\delta} - 1\right) + \frac{1}{r+\delta}. \tag{45}$$

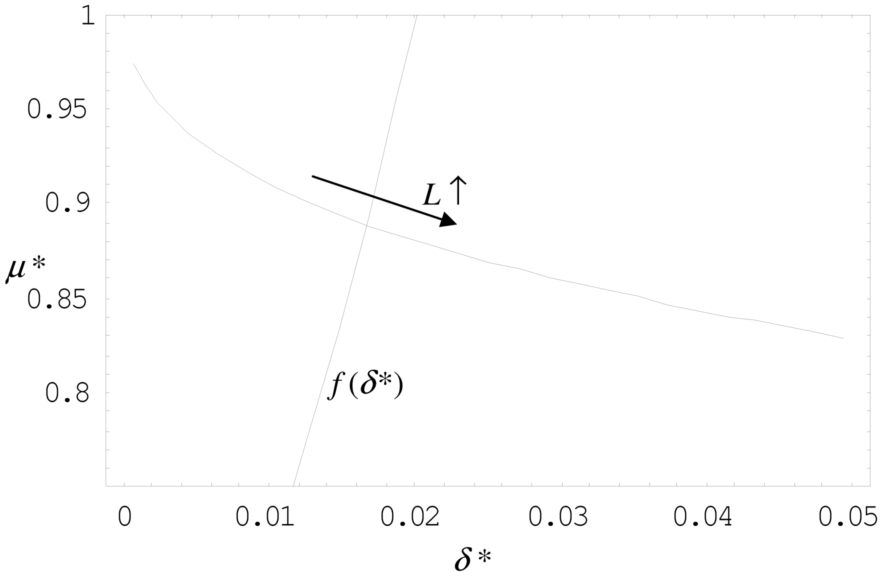

Figure 8: The values of parameters are $\lambda = 1.5$, $\beta = 0.95$, $a = 10^{-8}$.

Note that, since $\gamma_Y^*(\delta)/(\ln\lambda) = (1-\mu^*(\delta))G(n^*(\delta)) = \delta\mu^*(\delta)$ is maximized at $\delta = \delta^{\max}$, it must be true that:

$$\frac{d\mu^*(\delta^{\max})}{d\delta} = -\frac{\mu^*(\delta^{\max})}{\delta^{\max}}. \tag{46}$$

Substituting (45) and (46) into (44) gives:

$$\begin{aligned}\left.\frac{dU_t^*(\delta)}{d\delta}\right|_{\delta=\delta^{\max}} =& \frac{1}{(1-\beta)\delta^{\max}}\left[(\ln\lambda)\mu^*(\delta^{\max}) - \frac{r\pi\mu^*(\delta^{\max})+\delta^{\max}}{r+\delta^{\max}-r\pi\mu^*(\delta^{\max})} + \frac{\delta^{\max}}{r+\delta^{\max}}\right]\\ =& -\frac{(\ln\lambda)r\pi\mu^*(\delta^{\max})}{(1-\beta)\delta^{\max}\left(r+\delta^{\max}-r\pi\mu^*(\delta^{\max})\right)}\cdot\\ &\left[\mu^*(\delta^{\max}) - \frac{r+\delta^{\max}}{r\pi} - \frac{1}{\ln\lambda}\left(\frac{r}{r+\delta^{\max}}-2\right)\right].\end{aligned} \tag{47}$$

A marginal increase in δ from $\delta^{\max}$ (i.e., a marginal relaxation of IPR protection from the growth-maximizing level) is welfare improving if and only if expression (47) is positive. From (47), it turns out that this is the case if and only if:

$$\mu^*(\delta^{\max}) < f(\delta^{\max}) \equiv \frac{r+\delta^{\max}}{\pi r} + \frac{1}{\ln\lambda}\left(\frac{r}{r+\delta^{\max}}-2\right). \tag{48}$$

It is straightforward to confirm that the value of $f(\delta^{\max})$ is positive and that it takes a larger value when $\delta^{\max}$ is larger. Figure 8 depicts a representative shape of $f(\delta^{\max})$ in (δ, μ) space. A marginal increase in δ from $\delta^{\max}$ is beneficial for welfare if and only if the pair $(\delta^{\max},\ \mu^*(\delta^{\max}))$ is below the curve of $f(\delta^{\max})$.

Note that the position of the curve of $f(\delta^{\max})$ only depends on β and λ (recall that $r = (1-\beta)/\beta$ and $\pi = (\lambda - 1)/\lambda$), whereas the $(\delta^{\max},\ \mu^*(\delta^{\max}))$ pair can change depending on other parameters. For example, as depicted in Figure 7, the growth-maximizing level of IPR protection $(\delta^{\max})$ changes significantly with the size of population L. The downward-sloping curve in Figure 8 depicts the locus of $(\delta^{\max},\ \mu^*(\delta^{\max}))$ as L is gradually increased. This numerical example shows that relaxation of IPR protection is beneficial whenever the economy has a reasonably large population such that the growth-maximizing level of IPR protection allows more than around 1.5 % of state-of-the-art goods to be imitated within a period. For a reasonable range of parameters, we confirmed that $\mu^*(\delta^{\max}) < f(\delta^{\max})$ is likely to hold unless $\delta^{\max}$ is quite small.

A.2 Endogenous Imitation

In the text, it is assumed that the authority can directly choose the probability of imitation. In this appendix, we discuss how the behavior of the economy changes if the authority can choose the difficulty of imitation by outsiders, and therefore the probability of imitation is determined endogenously.

Suppose that, in a monopolized sector, outsiders may try to imitate the incumbent monopolist's good at the beginning of each period. A worker who attempts this succeeds with probability c, and fails with probability $1 - c$. If he (or she) fails, he can work neither as a production worker nor a researcher. If he succeeds, he can hire production workers and produce the imitated good at the marginal cost of w_t (the same as the monopolist). We assume that, when some outsider successfully imitates the state-of-the-art good, only the original monopolist and successful imitator can produce goods. However, from the next period onwards, all outsiders can learn costlessly how to imitate. (In reality, imitated goods are unlikely to be protected by patents.)

In this setting, the incentives of imitators derive only from the profit that can be obtained in the period when they imitate successfully. In addition, the amount of this profit varies with the number of successful imitators, since it is possible that more than one imitator can succeed simultaneously. For simplicity, we allow the original monopolist and successful imitators to collude so that the monopoly price is maintained and the monopoly profit is divided equally among them. Then, if k imitators succeed simultaneously, each successful imitator's payoff is $\pi/(1+k)$.

The expected profit from imitative activity is calculated as follows. Let l denote the number of outsiders who are trying to imitate the monopolist's good. Then, the probability that k of them succeed is given by the binomial distribution:

$$\frac{l!}{k!(l-k)!}c^k(1-c)^{l-k}.$$

Conditional upon this, the probability that one imitator succeeds is k/l. Therefore, the expected profit from imitative activity is given by:

$$\rho(l,c) \equiv \sum_{k=1}^{l} \frac{\pi}{1+k}\frac{k}{l}\frac{l!}{k!(l-k)!}c^k(1-c)^{l-k},$$

which is decreasing in l and increasing in c. Likewise, the expected profit of the incumbent monopolist for one period is given by:

$$\widetilde{\pi}(l,c) \equiv \sum_{k=0}^{l} \frac{\pi}{1+k}\frac{k}{l}\frac{l!}{k!(l-k)!}c^k(1-c)^{l-k}.$$

Observe that the monopolist earns the largest profit when $k=0$, whereas imitators can earn profits only when $k \geq 1$.

The number of imitators is determined by the free entry condition:

$$\rho(l,c) \leq w \quad \text{with equality if } l > 0,$$

which shows that l is determined by c and w so that we can write $l = l(c,w)$. From the property of $\rho(\cdot)$, $l(c,w)$ is increasing in c and decreasing in w. Those results also hold for the endogenous rate of imitation, $\delta = 1 - (1-c)^l$. Therefore, we can write:

$$\delta = 1 - (1-c)^{l(c,w)} = D(c,w), \tag{49}$$

where function $D(\cdot)$ is increasing in c and decreasing in w.

Suppose that the authority can choose the ease of imitation c. Now consider how c should be set when the authority wants to achieve a desired rate of imitation δ. By inverting equation (49), we obtain:

$$c = C(\delta, w), \tag{50}$$

where $C(\cdot)$ is increasing in both δ and w. That is, imitation must be easier when the authority desires a higher rate of imitation, or when the equilibrium wage (i.e., the opportunity cost of imitative activity) is higher. Using (50), the number of imitators can be written in terms of δ and w:

$$\widetilde{l}(\delta, w) = l(C(\delta, w), w).$$

$\widetilde{l}(\cdot)$ is increasing in δ, but the sign of $\widetilde{l}_w$ is ambiguous.

The remaining analysis proceeds in essentially the same way as in Section 5. Recall that the BGP is characterized by (20)-(23). Now profit π in (21) and (22) must be replaced by $\widetilde{\pi}(\widetilde{l}(\delta, w^*(z,\delta)), C(\delta, w^*(z,\delta)))$. In addition, we must subtract the aggregate number of imitators $\mu^*(z,\delta)\widetilde{l}(\delta, w^*(z,\delta))$ from the denominator of (22). This modified system is solvable in principle since the number of unknown variables is the same as in the original system. Although we cannot obtain the explicit solution, it can be expected that the solution would be qualitatively similar to that obtained in Section 5, given that the significance of above modifications in the aggregate economy is not large.